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As to Policies, dated after 25th March 1860, which receive Five-sixths of the Profits.

It was resolved that all Assurers who effected Policies in the Participating Class, after the 25th March 1860, should receive *five-sixths* of the entire profits to arise from all classes of the Assurances completed after that date. Of the Assured who have joined the Company since the 25th March, 1860, there are 1,738 whose Policies are in the Participation Class, and were in force on the 25th March 1865, and who accordingly shared in the Profits at this Division for the first time. The Bonuses which have been allotted to their Policies are stated in the following Table; and, for comparison, the Bonuses that were declared on Policies of like duration and amount at the last Division in 1860 are also stated:—

TABLE OF BONUS ADDITIONS

To Policies effected since the date of the last Division of Profits on the 25th March 1860.

Policies effected before 25th March	Number of Premiums paid.	Sum Assured.	**Bonus, 1865.**	Bonus, 1860
1861	5	£1000	£68	£60
1862	4	£1000	£54	£48
1863	3	£1000	£41	£36
1864	2	£1000	£27	£24
1865	1	£1000	£14	£12

The Bonuses allotted at this Division, to Policies which have been effected in the last five years, are therefore 13 per cent. *larger* than the Bonuses allotted, at the last Division in 1860, to Policies of equal duration and amount.

PRACTICAL QUESTIONS.

SUGGESTIONS have been made at various times that the usefulness and interest of this *Journal* might be much increased if greater prominence were given to questions of a practical character; and it has been suggested in particular, that solutions might with advantage be inserted of some of the more difficult cases that come before actuaries in the course of their practice. We are therefore induced to give the particulars of the two following cases, with the approximate solutions which appeared sufficiently accurate to the actuary to whom the questions were submitted. We shall be glad if such of our readers as may have leisure will give more exact solutions; and we shall also be glad to receive other approximate solutions from persons who may consider the following unsatisfactory.

Question 1. "The sums of £80,000 Consols and £8,000 New Three per Cent Stock, are settled on the following trusts:—

"An annuity of £500 is to be paid to a gentleman, A, aged 75, for his life; and on his death a sum of £1000 is to be paid to each of his five children. Three annuities of £250 each are to be paid to three ladies, B, C, D, aged 80, 76, and 71 for their respective lives; and on the death of B, £1000 is to be paid to each of her three children; and on the death of D, £1000 to each of her four children. Subject to these payments, the surplus income of the Trust Fund is to be invested in similar stock as received, and to accumulate until the death of the last survivor of the four annuitants, when the capital is to be divided.

"X is absolutely entitled on the death of such last survivor to one 20th part of the accumulated sum, and to one 7th of another 20th part; and wishes to know what sum he can borrow on security of his share."

The present annual income of the fund being £2640, ($=3\times 880$) and the annuities amounting to £1250, there is a present annual surplus of £1390; or rather less, when income tax is taken off; and the total amount of the legacies to be paid is £12,000.

Consider first what amount of stock will suffice to pay these legacies. Taking stock at the mean minimum price of 85, it would require £14,117·65 Stock to pay the legacies of £12,000; and the annual income of this stock (£420·529) will be available towards payment of the annuities to A, B, and D.

We first then set apart the above amount of stock—£14,117·65,—for payment of the legacies, and part payment of the annuities; and we then seek what further amount of stock must be set apart to provide for payment of the balance of the annuities. We must consider separately the cases of A B C and D. On the death of A (age 75) there is payable £5000, or say £5882·36 Stock, the annual income from which is £176·471. The balance of the annuity is £323·529 (=£500−£176·471). On the death of B (age 80) £3000 is payable—say £3529·41 Stock—the income of which is £105·882; and the balance of B's annuity is therefore £144·118. The annuity for the life of C (age 76) is £250. On the death of D (age 71) £4000 is payable, or say £4705·88 Stock—the income of which is £151·176; and the balance of D's annuity is £108·824.

We have therefore to calculate the amount of stock that will provide for the following annuities.

£323·529	on life of 75	the value of which according to the Carlisle 3½ per cent tables is found to be	1899·9
144·118	,, 80	,, ,,	687·7
250·000	,, 76	,, ,,	1411·9
108·824	,, 71	,, ,,	766·4
826·471			4765·9

Since we take stock at 85, we may assume that £100 sterling produces interest at the rate of 3·529 $\left(=\frac{3}{\cdot 85}\right)$ per cent per annum; or we may value the above annuities at 3½ per cent interest. Taking then the values of $a_{75}+\cdot 5$, $a_{80}+\cdot 5$ &c. by the Carlisle 3½ per cent tables, we get the values of the annuities as set out above—the total value being £4765·9 sterling, or, say, £5606·9 Stock. We thus conclude that this amount of stock will provide for the balance of the annuities. Adding to this, the amount obtained above, £14,117·6, we conclude that a sum of £19,724·5 Stock will provide for payment of all the annuities and legacies. This leaves a sum of £68,275·5 Stock, which is to accumulate at compound interest until the death of the last survivor of A, B, C, D.

We have now to investigate a formula applicable to this case. Suppose that £1 is invested at compound interest at the rate j until the termination of a given status, so that if the status terminate in the 1st, 2nd, 3rd, . . . , nth, years respectively, there will be receivable the sums, $1+j$, $(1+j)^2$, $(1+j)^3$, ... $(1+j)^n$; and let the value of the reversion be estimated at the rate of interest i; then if q_1, q_2, . . . q_n are the probabilities of the status terminating in the 1st, 2nd, . . . nth, years, the value of the reversion will be

$$q_1\frac{1+j}{1+i}+q_2\frac{(1+j)^2}{(1+i)^2}+\ \ldots\ +q_n\frac{(1+j)^n}{(1+i)^n}+\ \ldots\ .$$

But this is the value of £1 to be received at the termination of the given status, estimated at a rate of interest I, such that

$$\frac{1}{1+\mathrm{I}}=\frac{1+j}{1+i};\ \text{or}\ \mathrm{I}=\frac{i-j}{1+j}.$$

Now to apply this formula to the case under consideration, we must first determine at what rate of interest the stock may be considered to accumulate. Each £100 stock produces annually £3 *sterling* interest, which is to be invested, say at the price 95, and will therefore produce $\frac{3}{\cdot 95}=3{\cdot}158$ *Stock*. If then we make $j={\cdot}03158$, and $\mathrm{I}={\cdot}03$ (because this is the rate at which we have tables calculated) we shall have

$$i=(1+\mathrm{I})(1+j)-1={\cdot}0625$$

and we shall estimate the value of the reversion at $6\frac{1}{4}$ per cent—a very fair rate. We have thus next to find the value (by the Carlisle 3 per cent tables) of the reversion to £1 on the death of the last survivor of A, B, C, D; or 80, 76, 75, 71. Thus

$$\begin{array}{rl@{\qquad\qquad}rl}
a_{80}= & 4{\cdot}365 & a_{75}= & 5{\cdot}512 \\
a_{76}= & \underline{5{\cdot}277} & a_{71}= & \underline{6{\cdot}737} \\
 & 9{\cdot}642 & & 12{\cdot}249 \\
a_{80.76}= & \underline{2{\cdot}728} & a_{75.71}= & \underline{3{\cdot}672} \\
a_{\overline{80\cdot76}}= & 6{\cdot}914=a_{71} & a_{\overline{75\cdot71}}= & 8{\cdot}577=a_{66} \\
a_{\overline{75\cdot71}}= & \underline{8{\cdot}577}=a_{66} & & \\
 & 15{\cdot}491 & & \\
a_{71.66}= & \underline{4{\cdot}882} & & \\
a_{\overline{80\cdot76\cdot75\cdot71}}= & 10{\cdot}609 & & \\
\mathrm{A}_{\overline{80\cdot76\cdot75\cdot71}}= & {\cdot}66188 & &
\end{array}$$

Hence the value of the reversion to £68,275·5 Stock expectant on the death of the last survivor is £45,190 stock; or £38,411·5 sterling, if we again take stock at 85, since it is now a question as to the selling price. Lastly, the share of X, ($\frac{1}{20}$th $+\frac{1}{7}$ of $\frac{1}{20}$th) will be of the value £2195.

It must be borne in mind that the share of X is probably subject to legacy or succession duty; and that a double duty is probably payable on the $\frac{1}{7}$th of the $\frac{1}{20}$th share. Nor must it be lost sight of that accumulation for more than 21 years after the death of a testator is illegal. Altogether, we may conclude that unless the legacy duty is very heavy, the reversion might be expected to sell for about £2000.

INSTITUTE OF ACTUARIES.

SOLUTIONS OF THE SECOND YEAR'S EXAMINATION QUESTIONS.

(Continued from vol. xiii., p. 260.)

Questions for 1864.

1. Find an expression for the Napierian logarithm of any number in terms of the logarithm of the preceding number; and by the aid of such expression find $\log_{10}2$ to five decimal places.

2. To what base is $-1·765776$ the log. of 1,000? Given reciprocal of $1·765776=·566323$.

Ans. Base $=\dfrac{1}{50}$.

3. Find the probability of turning up 1, 2, 5, in any order, in three successive throws of a die. Explain the principles assumed in the solution of the question.

Ans. $\dfrac{1}{36}$.

4. A and B play together as follows:—A throws a coin and continues to throw as long as he throws head. If this happen four times, he wins; if not, he resigns the coin to B on the same terms, B to resign it again if he should fail, and so on. The first who throws four heads in succession wins one shilling. Find the values of the expectations of A and B.

Ans. $\dfrac{64}{127}$ and $\dfrac{63}{127}$.

5. Find the relation between the discount of any sum payable n years hence, and the present value of an annuity for the same term of years. Distinguish between the discount here referred to and commercial discount, as well as discount at simple interest.

Ans. If D be the discount, A the annuity, $\mathrm{A}=\dfrac{\mathrm{D}}{i}$.

6. Construct a specimen table of the present value of an annuity, at 3 per cent. interest, of £1 for 25, 26, &c., years, to 30 years. Given $v^{30}=·4120$.

7. Give some account of the Carlisle table of mortality. What is to be remarked as to the relative proportion of the sexes among the lives on